Brief Communication

Comment on "Multiport Representation of Inertia Properties of Kinematic Mechanisms"†

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Allen discusses a technique for generating Lagrange's equations directly from a bond graph. The essential step is the transformation of a physical inertia (I) over a (displacement) modulated transformer (T) representing a mechanism, into a virtual inertia ($\tilde{I} = T^T I T$) and a so-called gyristor ($G = T^T I \dot{T}$). The inertias are expressed in Lagrangian variables q, $\dot{q}$ in contrast with the Hamiltonian form (p, q) used by Karnopp and Rosenberg (**1**) (Fig. 1).

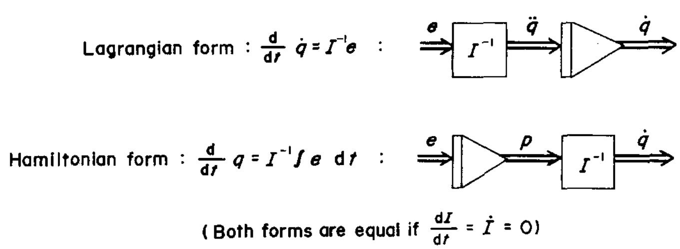

FIG. 1. Inertia definitions for Lagrangian and Hamiltonian variables respectively.

In an energy analysis (p. 246) the kinetic coenergy of the physical inertia is expressed in terms of the virtual inertia. From this expression (42) is derived:

$$E_k^* = \frac{1}{2}\dot{q}_{kI}^T[G^T - G]\dot{q}_{kI} = 0. \tag{42}$$

Using (42) Allen presumes numerical advantages because he concludes from (42) the matrix G to be symmetric.

Our comment is that the use of this symmetry in computations may lead to considerable errors, because the conclusion that G is symmetric is essentially wrong. A simple way to demonstrate this is by means of matrix algebra:
(a) It is well known that a quadratic form of an antisymmetric matrix is always zero: if

$$M_a = -M_a^T$$

†By R. R. Allen, *J. Franklin Inst.*, Vol. 308, pp. 235–253, 1979.

then

$$x^T M_a x = 0.$$

(b) Any matrix M can be decomposed into a symmetric part M_s and an antisymmetric part M_a:

$$M = \tfrac{1}{2}(M + M^T) + \tfrac{1}{2}(M - M^T) = M_s + M_a.$$

So $\frac{1}{2}(G - G^T)$ is the antisymmetric part of G.

In Allen's case has been derived

$$\dot{q}_{kI}^T \left[\frac{1}{2}(G - G^T) \right] \dot{q}_{kI} = 0 \tag{42}$$

which according to (a) means that G can still be any matrix.

A physical interpretation of this mathematical fact is that the antisymmetric part of the Onsager matrix (phenomenological relationship between the flow- and effort vector) does not contribute to the entropy production or to the dissipation of free energy.

To emphasize that G is not necessarily symmetric, we recall a well-known example from 3-D mechanics. The case to be considered is the transformation of an inertia from a rotating coordinate system into an inertial coordinate system having the same origin. This transformation results in an "Eulerian Junction Structure" or EJS (**2**), so-called because it follows from Euler's equations. The EJS is characterized by an antisymmetric matrix as can be easily seen by writing the equations in matrix form. Using Allen's terminology this EJS can be considered as a gyristor with an antisymmetric gyristance.

As noted already, the essential step in Allen's paper is the transformation of a physical inertia over a modulated transformer into a virtual inertia and a gyristor, using the Lagrangian form of the inertias. This technique itself is not affected by our comment. It can even be extended to the Hamiltonian form of the inertias (cp. Fig. 1.) This results in a slightly different gyristance, G_H:

$$G_H = -\dot{T}^T I T = -(T^T I \dot{T})^T = -G^T.$$

[$\stackrel{*}{=}$ I is symmetric ($I = I^T$) because of the Maxwell reciprocity relations.]

In case the virtual inertia is constant ($\dot{I} = 0$), the kinetic energy and coenergy are equal. This means that the Hamiltonian and Lagrangian approach have to result in identical gyristors, so

$$G = G_H = -G^T$$

which is the mathematical definition of an antisymmetric matrix. The EJS is an example of this situation.

References

(**1**) D. C. Karnopp and R. C. Rosenberg, "System Dynamics: A Unified Approach", John Wiley, New York, 1975.

(**2**) D. C. Karnopp, "The energetic structure of multibody dynamic systems", *J. Franklin Inst.*, Vol. 306, pp. 165–181, 1978.